The Priorities and Challenges of Primary Teachers' Knowledge in their Mathematics Planning.

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There is growing consensus that the process of planning mathematics lessons is as complex as teaching them, yet there is limited research on this. This paper reports on one aspect of a project examining issues in primary teachers' mathematics planning. The results, taken from a questionnaire completed by 62 primary teachers, indicate that when planning their lessons, teachers give priority to a diverse range of aspects related to their mathematical knowledge for teaching, yet there are similarities in the challenges which they experience. Findings also suggest that team planning can support teachers overcome such challenges. Issues requiring further attention are discussed.

Planning is a critical part of the teaching process and studies have emphasised the complexities and varied approaches teachers use to develop their plans for mathematics teaching (e.g. Sullivan, Clarke, Clarke, Farrell, & Gerrard, 2013). Further to this, it has been argued that the decisions teachers make when planning have the power to directly impact student thinking and learning about mathematics (Kilpatrick, Swafford, & Findell, 2001).

One of the main functions of planning is often described as the time allocated to selecting, preparing and designing activities for students (Kilpatrick et al., 2001). Yet compared to their international counterparts, researchers have considered the planning of Australian mathematics teachers as unique and complex (Sullivan, Clarke, Clarke, Gould, Leigh-Lancaster, & Lewis, 2012b). For example, it has been suggested that primary teachers in the US generally rely on instructional materials such as textbooks as the springboard for their planning. Teachers then select tasks within these materials and make decisions on how they will organise such tasks (Kilpatrick et al., 2001). This is in contrast to Australia where a common approach appears to be for primary teachers to develop mathematical units of work as a team. These teachers often use a combination of official curriculum documents, self and team-developed resources, teacher assessments and web-based materials to guide their mathematics planning (Clarke, Clarke, & Sullivan, 2012a).

Parallel to this is the link between effective teaching and student outcomes (Askew, Brown, Rhodes, William, & Johnson, 1997). In describing effective teachers of mathematics, the then Victorian Department of Education and Early Childhood Development (2010) urged teachers to consider elements such as curriculum, student interests, pedagogy, assessment and differentiation in their mathematics planning, yet there is limited advice for teachers on the ways such planning may be carried out. In my experience, as a primary school leader and teacher, primary teachers often report difficulties in planning mathematical experiences which encompass the complexities of mathematics teaching including, but not limited to, curriculum requirements and catering to the wide range of their students' knowledge, skills and understandings. As part of a larger project I am seeking to investigate the challenges facing primary teachers in their mathematics planning in order to optimise the effectiveness of teachers' planning processes for the benefit of teachers and students alike.

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The following outlines the literature informing the perspective on teacher planning that underpins this research. The subsequent sections describe the conceptual framework which guided the data collection and the context for the data collection. The results, discussion and conclusion are then presented.

Teachers' Mathematical Knowledge for Teaching and Planning

An assumption underpinning the research reported below is that effective teaching is preceded by effective planning. That is, teachers are best able to cater for students' needs when they have a clear vision of what they want their students to learn and how they will come to learn it (Hattie & Timperley, 2007). More specifically, teachers' planning decisions are informed by their mathematical knowledge for teaching.

Ball, Thames, and Phelps, (2008) offer a schematic representation of the knowledge used by teachers for the teaching of mathematics. It includes two major categories: subject-matter knowledge (SMK) and pedagogical content knowledge (PCK). In this model, SMK includes the mathematics which is used in settings other than teaching such as understanding that the number two is even as well as the mathematics that is unique to teaching, such as being able to identify patterns in student errors. SMK also includes appreciation of the interconnectedness of mathematical ideas, including those found in the curriculum. The category of PCK includes knowledge of the 'teaching' of mathematics such as making decisions on the most effective model for teaching a particular concept. The domain also includes being able to anticipate ways in which students will respond to a task including familiarity with the common misconceptions students are likely to experience, as well as teachers' understanding of curriculum documents, for instance, being able to identify the big ideas of what they are about to teach. In line with Ball, Thames, and Phelps (2008), the term 'mathematical knowledge for teaching' will be used to refer to the domains of SMK and PCK collectively.

One aspect of teachers' mathematical knowledge for teaching is the planning decisions teachers make in selecting the tasks they will teach (Ball, Thames, & Phelps, 2008). For example, in a research synthesis, Anthony and Walshaw (2009) concluded that "in the mathematics classroom, it is through tasks, more than in any other way, that opportunities to learn are made available to the students" (p. 96). Yet it appears the parameters for selecting such tasks and teachers' capacity to decide on the relevance of tasks is varied (Sullivan, Clarke, & Clarke, 2012a). It has also been found that some teachers experience difficulties in articulating the 'big ideas' which inform their teaching (Clarke, Clarke, & Sullivan, 2012b), an inference being that this will also impact on teachers selection and use of appropriate tasks.

Connected to the way teachers' mathematical knowledge for teaching influences planning decisions is the suggestion that teachers may feel constrained to teach in certain ways or use particular tasks if they anticipate negative student responses such as a lack of persistence or risk-taking (Sullivan, Walker, Borcek, & Rennie, 2015). In response to these anticipated reactions, it has been proposed that teachers may reduce the cognitive demands of the task during planning and over explain content during lessons (Sullivan et al., 2015). This has implications for classroom culture (Rollard, 2012) and may also have ramifications for student learning and dispositions (Dweck, 2000). According to Rollard (2012), teachers can take purposeful actions to develop mastery orientations in their students where the learning of content rather than competitive performance is valued. In turn, this leads to a

positive classroom culture which results in increased student effort and improved student outcomes.

The focus of this paper is to review aspects of a baseline questionnaire completed by practising teachers to explore some of the issues Australian primary mathematics teachers experience in their lesson planning. The results reported below are intended to offer insights about the following research questions:

- How does teachers' mathematical knowledge for teaching influence and inform their planning of mathematics lessons?
- What are the priorities and challenges for teachers in planning their mathematics lessons?

Context for Data Collection and Methodology

The data which is the focus of this paper was collected as part of the Encouraging Persistence, Maintaining Challenge project (Sullivan et al., 2015) which is investigating a particular planning sequence and lesson structure which aims to encourage students to persist on unfamiliar tasks in order to build mathematical connections for themselves. The aspect of the research reported on in this paper explores teacher planning as one of the key issues.

The overall project is informed by a conceptual framework which proposes that teacher planning and subsequent classroom actions are a function of their beliefs about mathematics, their knowledge about mathematics and pedagogy and the constraints which teachers believe may be encountered when teaching a lesson (Sullivan et al., 2015). The focus of the data presented below is on the aspect of the model which examines the connection between teachers' mathematical knowledge for teaching and their planning decisions: the inference being that in order to effectively support their students, teachers need to understand curriculum expectations as well as the concepts they are teaching and how they are going to teach them.

The following data were collected through an online questionnaire designed by the author, which was completed by 62 Victorian primary teachers who attended a professional learning day as part of the project. The respondents were teachers of students from Years 4 to 6 (ages 9-12) who together, represented 25 different schools with students from a range of socio-economic backgrounds. It was also noted that the teachers had a variety of experience - many of the teachers who attended the professional learning day were experienced teachers who had a leadership role in the school (e.g., team leader, mathematics curriculum leader) or had an interest in the teaching of mathematics, whereas others were in their first five years of teaching. It is worth noting that it is unusual to get responses from such a large and diverse group of practicing teachers and the data should therefore provide valuable insights into the nature of teachers' mathematics planning. Because the intent of the questionnaire was to gather baseline data about teachers' planning priorities and to seek insights into their perceptions about their planning overall, items were constructed to allow for the comparison of different aspects of teacher planning, specifically their mathematical knowledge for teaching. Additionally, in order to preserve the anonymity of respondents, no background information was sought.

Teachers were provided with approximately 30 minutes to complete the survey at the commencement of the professional learning day. The survey comprised of both fixed Likert scale type items and free format questions to which teachers could respond. In analysing the

data, the intent was to summarise and describe (Creswell, 2007) the teachers' responses as a way of answering the research questions. In doing so, fixed items were analysed with the number of teachers presented as raw figures and mean responses represented below. Open responses were coded to identify emerging themes. The research questions and conceptual framework were used to frame the analysis for both fixed and open items.

Results

The results are reported in two sections: the first being responses to fixed items, the second section being teacher responses to an open item. One of the survey items asked teachers to imagine they had a unit plan and were now developing a mathematics lesson to teach. Teachers were then presented with a list of 21 items which reflected the different aspects of mathematical knowledge for teaching. Teachers were asked to 'drag and drop' the five most important items they considered when planning for that lesson.

Table 1
Top 5 Factors Teachers Consider when Planning Individual Mathematics Lessons (n=62)

Item	No. of teachers
How I can extend students on the given task	36
How I can support students who may experience difficulties	35
The questions I will ask students to promote mathematical thinking	34
How well I understand the content of what I'm about to teach	26
How students will record and share their thinking	26
The fit with my curriculum goals	20
How I will communicate the main ideas of the lesson with the students	20
The usefulness of the task	20
The place of the lesson in a sequence	13
How students will reflect at the end of the lesson	13
Formal student assessment data	12
Student groupings	10
How the students will interact during the lesson	10
Making sure the students have fun	10
Ensuring there is a balance of lesson types over time	8
How the lesson will be introduced	7
Whether I have anticipated student responses	4
My own teacher judgments about students	3
Whether I have worked through the task	1
Whether I have taught a similar/the same lesson previously	1
The physical classroom environment set up	0

In doing so, teachers were required to make a quantifiable judgement about their priorities in mathematics planning, an inference being that these results provide insight into

the priorities teachers give to aspects of their mathematics planning, particularly their mathematical knowledge for teaching. Table 1 presents the responses to those items. In other words, 36 out of 62 teachers selected how they can extend students as one of the top 5 factors they consider in their planning. The first comment is that the top three factors relate to students, which can be taken to mean that when planning, teachers place high importance on addressing students' needs in the classroom. A second comment is that the top 5 responses reflect various aspects of both teachers' SMK and PCK. Additionally, that teachers give priority to extending and supporting students can be taken to mean that differentiation is high on teachers' planning agendas. Furthermore, the importance given to the consideration of tasks, understanding content, student questioning and discussion has implications for the competencies of teachers' mathematical knowledge for teaching and the impact such knowledge may bring to bear on subsequent planning decisions and the effectiveness of lessons.

A last comment, is that responses amongst the factors were diverse and illustrate the range of priorities among teachers in their mathematics planning. Items chosen only a few times are also of interest as many would argue that these are also important aspects of mathematics planning and highlight the idiosyncrasies and the diverse priorities of teachers' mathematics planning, especially those related to teachers' mathematical knowledge for teaching.

A following prompt on the questionnaire explored whether there was something else that was important to the teacher that was not on the list. Many teachers commented on student engagement, for example:

> Making lessons engaging, explicit, encouraging and encouraging a growth mindset.

The consideration given to resourcing was also noted in comments such as:

> Having resources to support and scaffold for a variety of learners.

There were also teachers who made comments about the complexities involved in their daily mathematics planning, such as:

> I try to consider all of these things when planning, choosing a top 5 is difficult.

While teachers identified the aspects of their planning they deemed most critical, it was also important to explore how teachers felt about those aspects of their planning – that is, how confident they feel about planning for certain elements of their mathematics lessons, in order to provide insights into potential issues teachers experience in their mathematics planning.

In doing so, a second survey item asked teachers to indicate their agreement with statements in relation to their planning and teaching of mathematics. Teachers rated each statement from strongly disagree (score 1) to strongly agree (score 5). Whilst acknowledging that this is not an interval scale, the mean score is presented to allow for comparison between the strength of agreement with items.

In interpreting the tables, note that a mean score of 3 would indicate that the responses were evenly spread around "neither agree nor disagree" and a score of 3.5 would indicate that the responses were, on balance, half undecided and half agreeing. Table 2 presents the mean responses of teachers to statements about their mathematical knowledge for teaching.

The results in Table 2 show similar agreement amongst the items and highlight the spread in teachers' perceptions about their capacity to plan mathematics effectively. It is also noted that of the 36 teachers who ranked extending students in a given task in their top 5

priorities, 14 of them felt that they lacked the ability to sufficiently do so. Additionally, out of the 35 teachers who ranked supporting students on a given task in their top 5 priorities, 9 teachers felt they did not have sufficient knowledge to do so. Likewise, out of the 26 teachers who rated understanding content in their top 5 priorities, 9 teachers felt that their knowledge of mathematics was lacking - an inference being that while some teachers may want to plan and teach in a certain way, that their mathematical knowledge for teaching may prevent them from doing so. It is noted that the three items compared above represent the responses of 23 individual teachers.

Table 2
Teacher Responses to Statements about their Mathematical Knowledge for Teaching (n=62)

Statement	Mean Score
My knowledge of mathematics is good enough that I can plan whatever types of lessons I like for this level.	3.66
My knowledge of ways of teaching mathematics is good enough that I can plan whatever types of lessons I like for this level.	3.53
I know ways of catering for students who experience difficulties with mathematics.	3.73
I know ways of catering to students who require extension in mathematics.	3.55

Interestingly, the responses that rated very highly were those relating to teachers' planning processes and are presented in Table 3.

Table 3
Teacher Responses to Statements about their Mathematics Planning Processes (n=62)

Statement	Mean Score
I find it helpful to plan maths as a team.	4.51
I like to discuss my ideas for mathematics lessons with a colleague.	4.48
The way my team plans mathematics is helpful to my teaching.	3.98

The results in Table 3 are encouraging in the sense that they indicate teachers' willingness to collaborate for the purposes of their mathematics planning. On further inspection, it was found that of the 14 teachers who considered their knowledge to extend students on a given task was lacking even though it was in their top 5 priorities, 8 teachers agreed that that the way their team plans was helpful to their teaching. Additionally, of the 9 teachers who felt their knowledge to support students who experience difficulty was lacking, even though it was in their top 5 planning priorities, 5 of these teachers agreed that the way their team plans mathematics was helpful to their teaching. Lastly, of the 9 teachers who felt their knowledge of mathematics to plan whatever types of lessons they wanted was lacking even though understanding content was a top 5 planning priority, 6 teachers agreed that the way their team plans mathematics supported their teaching. Together, these responses represent 8 individual teachers. Overall, the data in Table 3 suggests that team planning has the potential to support teachers in their mathematics teaching, especially when they perceive their mathematical knowledge for teaching as requiring attention.

To gain further insight into the critical issues teachers face in their mathematics planning, respondents were invited to list the three biggest hurdles they needed to overcome.

All 62 participants responded, generating 178 items for analysis. Items were read and categorised according to emerging themes and related back to the theoretical framework and research questions. Where an item was coded in multiple categories that item was included in each category. For example, the comment 'Finding tasks that will meet the needs of all students within the group' met the criteria for SMK and PCK as well as being an indication of teacher beliefs and perceived constraints, which again illustrates the complexities involved when primary teachers plan their mathematics lessons. Representative responses as a result of this categorisation are presented below.

An overwhelming 60 out of the 62 respondents identified at least one, and in many cases two or more aspects of their mathematical knowledge for teaching as a hurdle to overcome. For example, comments coded as SMK, included:

> Feeling confident in my own mathematical knowledge to be able to extend students. Knowing actual working expectations of the year above - not just the curriculum expectations. Catering for a wide range of abilities.

Comments relating to PCK included:

> Anticipating student responses - the differences between thinking strategies that are used. Thinking about the right questions to ask students to prompt and deepen their thinking. Finding 'one' task that has different exit and entry points for ALL students.

In summary, these are thoughtful responses that provide insights into the challenges primary teachers experience when planning mathematics lessons. These comments emphasise that regardless of whether teachers give priority to their mathematical knowledge for teaching and how confident they feel in this aspect of their planning, that mathematical knowledge for teaching is at the forefront of teachers' minds when planning for mathematics instruction.

Discussion and Conclusion

The data reported above provide some insights into the priorities and challenges faced by primary teachers when planning. These data suggest that teachers' mathematical knowledge for teaching influences their planning decisions and that teachers' planning priorities are diverse and idiosyncratic.

In terms of the research questions, responses indicate that primary teachers' planning decisions are influenced and informed by their mathematical knowledge for teaching. In particular, responses illustrate that understanding content and differentiating the learning needs of their students weigh most on teachers' minds when planning their mathematics lessons. A surprising finding, however, was that despite the range of responses regarding how teachers felt about their mathematical knowledge for teaching, they reported a similar set of challenges: all but two teachers indicated aspects of their mathematical knowledge for teaching as hurdles to overcome. This has important implications for teacher educators.

In terms of the conjecture presented above, it would appear that if effective teaching is to be preceded by effective planning, that there are critical issues arising from the data which require further investigation. These include, for example, ways in which teachers can be supported in their understanding of learning pathways, knowing ways to select and modify tasks, familiarity with a variety of pedagogical approaches and ways to accommodate student diversity in the classroom. These issues are complex and pressing as they have the potential to influence classroom culture, student dispositions and outcomes (Rollard 2012; Dweck, 2000). Lastly, it is also important to consider the implications for professional

development in relation to team planning and how the team planning process can be utilised as a vehicle to support teachers in their mathematical knowledge for teaching and subsequent planning decisions.

The contribution of this paper is that the findings confirm mathematics lesson planning as complex and identify teachers' mathematical knowledge for teaching as a critical issue in teacher planning. Given the current priority schools are placing on teachers planning in teams and the value that teachers place on team planning, further research on this stage of planning would be useful in order to identify approaches that effectively support individuals and teams of teachers in overcoming the perceived hurdles of their mathematics planning.

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